

Fate of the false vacuum: towards realization with ultra-cold atoms

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Quantum decay of a relativistic scalar field from a false vacuum is a fundamental idea in quantum field theory. It is relevant to models of the early Universe, where the nucleation of bubbles gives rise to an inflationary universe and the creation of matter. Here we propose a laboratory test using an experimental model of an ultra-cold spinor Bose gas. A false vacuum for the relative phase of two spin components, serving as the unstable scalar field, is generated by means of a modulated radio-frequency coupling of the spin components. Numerical simulations demonstrate the spontaneous formation of true vacuum bubbles with realistic parameters and time-scales.

As proposed by Coleman in a seminal paper [1], the false vacuum is a metastable state of the relativistic scalar field that can decay by quantum tunneling, locally forming bubbles of true vacuum that expand at the speed of light. It has a close analogy with the ubiquitous phenomenon of bubble nucleation during a first order phase transition in condensed matter [2], e.g. the spontaneous creation of vapor bubbles in superheated water [3]. Applied to a quantum field such as the inflaton or Higgs field, bubble nucleation is an event of cosmological significance in some early universe models. Indeed, the Coleman decay scenario of the inflaton field features prominently in the theory of eternal inflation [4, 5], where bubbles continuously nucleating from a false vacuum grow into separate universes, each subsequently undergoing exponential growth of space [6]. This scenario, which could potentially explain the value of the cosmological constant by the anthropic principle, is currently being tested against observational evidence in astrophysical experiments [7, 8]. For an observer inside the bubble, the tunneling event — occurring in the observer’s past — appears like a cosmological “big-bang”, prior to inflation.

From a theoretical point of view, quantum tunneling from a false vacuum is a problem that can only be solved approximately [1, 9] (except for simplified models [10]) due to the exponential complexity of quantum field dynamics. This motivates the search for an analog quantum system that is accessible to experimental scrutiny, to test these models. The utility of such experiments, which complement astrophysical investigations, is that they would provide data that allow verification of widely used approximations inherent in current theories [11].

Here we demonstrate how to use an ultra-cold atomic two-component Bose-Einstein condensate (BEC) as a quantum simulator that generates a decaying, relativistic false vacuum. Quantum field dynamics occurs for the relative phase of two spin components that are linearly coupled by a radio-frequency field. In this proposal the speed of sound in the condensate models the speed of light, and the “universe” is less than a millimeter across.

Domains of true vacuum are observable using interferometric techniques [12] over millisecond time-scales with realistic parameters.

Modulating the radio-frequency coupling in time allows one to create a metastable vacuum from an otherwise unstable one [13] following Kapitza’s famous idea for stabilizing the unstable point of a pendulum by rocking the pivot point [14]. Our proposal requires repulsive intra-component interactions to dominate over inter-component interactions. To achieve this, we have identified a Feshbach resonance of ⁴¹K with a zero crossing for the inter-component *s*-wave scattering. Similar couplings in lower dimensions can also be achieved with a transverse double-well potential [15–17].

Previous work on ultra-cold atom analog models of the early universe has focused on the expansion of space-time [18, 19] and the formation of oscillons [20, 21]. While interesting cosmological analogs have been explored in liquid He [22, 23], its use as a quantum simulator is hampered by the limited tuneability of physical parameters and the phenomenological nature of available theoretical models. False vacuum decay models have been successfully applied to the quantum nucleation of phase transitions of liquid He [24, 25] in a non-relativistic context, but the nucleation and decay from a relativistic false vacuum has not yet been realized in a laboratory experiment. In this Letter we propose an implementation of Coleman’s model of quantum decay from a relativistic false vacuum with tuneable microscopic parameters. We simulate the quantum dynamics of the coupled Bose fields in the truncated Wigner approximation (TWA) [26, 27] and demonstrate how the resulting evolution can be imaged in one, two or three space dimensions using optical trapping. This shows the feasibility of a table-top experiment, and illustrates how the expected bubble nucleation dynamics depends on the dimensionality of space.

The dynamics of the scalar field ϕ in Coleman’s model [1] is given by the equation

$$\partial_t^2 \phi - c^2 \nabla^2 \phi = -\partial_\phi V(\phi), \quad (1)$$

where c is the speed of light, and the potential $V(\phi)$ has a metastable local minimum separated from a true vacuum by a barrier. We emulate this equation with a pseudo spin-1/2 BEC, where the speed of light is replaced by the speed of sound in the BEC, the relative phase between two spin components assumes the role of the scalar field ϕ , and the shape of the potential $V(\phi)$ is tunable. In addition there is an adjustable coupling to phonon degrees of freedom in our system, which serves to damp the dynamics. Our numerical simulations confirm the expected features of quantum tunneling dynamics with dissipation [28]. By addressing a radio-frequency transition between the spin components, the false vacuum initial state can be prepared and the final state read out by interferometry.

We consider a two-component BEC of atoms with mass m and a linear coupling ν realized by a radio-frequency field. Atoms with the same spin interact via a point-like potential with strength g . The Hamiltonian reads

$$\hat{H} = \int d\mathbf{r} \hat{\psi}_\sigma^\dagger(\mathbf{r}) \left[-\frac{\hbar^2 \nabla^2}{2m} - \mu \right] \hat{\psi}_\sigma(\mathbf{r}) - \nu \int d\mathbf{r} \hat{\psi}_\sigma^\dagger(\mathbf{r}) \hat{\psi}_{\bar{\sigma}}(\mathbf{r}) + \frac{g}{2} \int d\mathbf{r} \hat{\psi}_\sigma^\dagger(\mathbf{r}) \hat{\psi}_\sigma^\dagger(\mathbf{r}) \hat{\psi}_\sigma(\mathbf{r}) \hat{\psi}_\sigma(\mathbf{r}), \quad (2)$$

where summation over spin indices $\sigma \in \{-, +\}$ is implied. The Bose fields satisfy the usual commutation relations $[\hat{\psi}_\sigma(\mathbf{r}), \hat{\psi}_{\bar{\sigma}}^\dagger(\mathbf{r}')] = i\delta_{\sigma\bar{\sigma}}\delta(\mathbf{r} - \mathbf{r}')$. We introduce the quantum partition function $\mathcal{Z} = \int \mathcal{D}(\psi^*, \psi) e^{-S[\psi^*, \psi]}$ [29], where $S[\psi^*, \psi] = \int ds [\psi_\sigma^* \partial_\tau \psi_\sigma + H(\psi^*, \psi)]$. Here, $\mathbf{s} = (\tau, \mathbf{r})$ is a $d+1$ vector, $\tau = it/\hbar \in [0, \beta]$ is imaginary time, $\psi_\sigma(\tau, \mathbf{r})$ is a complex field subject to the periodic boundary condition $\psi_\sigma(\beta, \mathbf{r}) = \psi_\sigma(0, \mathbf{r})$. We look first for a static solution to identify vacua. This amounts to replacing $\psi_\sigma = \psi_0 = \text{const}$ in the saddle-point approximation $\delta S/\delta \psi_\sigma = 0$. For $\nu > 0$ we obtain the stable $|\psi_0|^2 = (\mu + \nu)/g$ and unstable $|\psi_0|^2 = (\mu - \nu)/g$ vacua with the two Bose gases being in phase and out-of-phase respectively. Let us introduce new field variables by $\psi_\sigma(\mathbf{s}) = \rho_\sigma^{1/2}(\mathbf{s}) e^{i\phi_\sigma(\mathbf{s})}$, where $\rho_\sigma(\mathbf{s}) = \rho_0 + \delta\rho_\sigma(\mathbf{s})$ and $\rho_0 = |\psi_0|^2$. The variables $\delta\rho_\sigma$ and ϕ_σ parametrize the deviation of the Bose fields from a vacuum. Substituting this parametrization into the action, we obtain

$$S(\rho, \phi) \approx \int ds \left[i\delta\rho_\sigma \partial_\tau \phi_\sigma + \frac{\hbar^2 \rho_0}{2m} (\nabla \phi_\sigma)^2 + \frac{(2g\rho_0 + \nu)\delta\rho_\sigma^2}{4\rho_0} + \frac{\hbar^2 (\nabla \delta\rho_\sigma)^2}{8m\rho_0} - \frac{\nu}{2\rho_0} \delta\rho_1 \delta\rho_2 - 2\nu\rho_0 \cos(\phi_a) - \nu \cos(\phi_a) \delta\rho_\sigma \right]. \quad (3)$$

Here $\phi_a = \phi_+ - \phi_-$ is the relative phase. Through elimination of slowly varying density fluctuations following the standard technique [29], an effective field theory

for the phase difference relevant for energies below $\hbar\xi c$ ($\xi = \hbar/\sqrt{2mg\rho_0}$ is the BEC healing length) is found

$$S(\phi_a) = \frac{1}{4g} \int ds [(\partial_\tau \phi_a)^2 + \hbar^2 c^2 (\nabla \phi_a)^2 + 2\hbar^2 V(\phi_a)] + \frac{4\nu^2}{g} \int ds \int ds' \cos \phi_a(\mathbf{s}) \cos \phi_a(\mathbf{s}') \mathcal{G}(\mathbf{s} - \mathbf{s}') \quad (4)$$

where the field potential $V(\phi_a) = -4\nu g \rho_0 \cos \phi_a (1 + \nu \cos \phi_a / 4g\rho_0) / \hbar^2$ and $c = \sqrt{g\rho_0/m}$ is the speed of sound. The non-local kernel $\mathcal{G}(\tau, \mathbf{r}) = (\beta V)^{-1} \sum_{\omega_n > 0} \sum_{\mathbf{k} > 0} e^{-i(\omega_n \tau + \mathbf{k} \cdot \mathbf{r})} \omega_n^2 / [\omega_n^2 + (c\hbar)^2 \mathbf{k}^2]$ is expressed through summation over Fourier momentum \mathbf{k} and Matsubara frequencies $\omega_n = 2\pi n/\beta$. The new action is similar to the one studied in Ref. [28], where the effect of dissipation on quantum dynamics was explored. Following Ref. [28] we obtain the following equation of motion for the relative phase:

$$\partial_t^2 \phi_a - c^2 \nabla^2 \phi_a + \frac{4\nu^2 \xi}{\hbar^2 c} \partial_t \phi_a = -\partial_{\phi_a} V(\phi_a). \quad (5)$$

It is similar to Eq. (1) with a new friction-like third term, due to coupling with density fluctuations. We note that in cosmological models such friction-like behavior may occur due to a homogeneous expansion of space, as commonly described by the Hubble constant [30].

By varying the tunnel coupling ν periodically in time, it is possible to alter the effective potential so that $\phi_a = \pi$ becomes a local minimum, which corresponds to a false vacuum in the sense of Coleman. We consider rapid oscillations of the tunnel coupling $\nu_t = \nu + \delta\hbar\omega \cos(\omega t)$, where the frequency of oscillations is $\omega \gg \omega_0 \equiv 2\sqrt{\nu g\rho_0}/\hbar$. Following Kapitza [14] the field ϕ_a may be viewed now as a superposition of a slow component ϕ_0 and rapid oscillations. Averaging rapid oscillations in time yields an equation of the form (5) with $\phi_a \rightarrow \phi_0$ and $V(\phi_a) \rightarrow V_{\text{eff}}(\phi_0) = -\omega_0^2 [\cos \phi_0 - 0.5\lambda^2 \sin^2 \phi_0]$. This potential, shown in Fig. 1b, develops a local minimum at $\phi_0 = \pi$ if $\lambda = \delta\hbar\omega_0/\sqrt{2\nu} > 1$, which corresponds to a false vacuum. Multiple equivalent true vacua occur at the global minima with $\phi_0 = 0, 2\pi, 4\pi, \dots$

We perform stochastic numerical simulations on the full BEC model (1) to investigate the bubble nucleation numerically and compare the results with the predictions of the effective theory developed above. The TWA, where a quantum state is represented by a stochastic phase space distribution of trajectories following the Gross-Pitaevskii equation [26, 27], enables one to simulate the entire experimental model of a three-dimensional coupled spinor BEC, in the limit of large occupation numbers per mode. This method is already known to accurately simulate BEC interferometric experiments down to the quantum noise level [12].

Our initial state construction then proceeds by assuming each mode is initially in a coherent state. The corresponding Wigner distribution is a Gaussian in phase

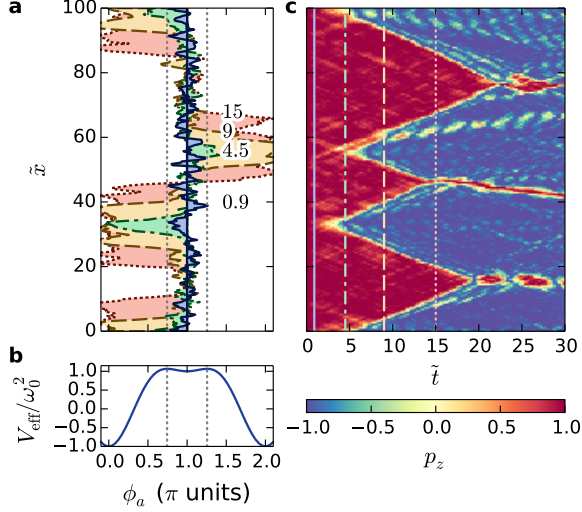


FIG. 1. Decay of the false vacuum in 1D. **a**, Single-trajectory simulation of the false vacuum decay in 1D with $N_{\text{grid}} = 256$ and dimensionless parameters $\lambda = 1.2$, $\tilde{\omega} = 50$, $\tilde{\nu} = 0.01$, $\tilde{\rho}_0 = 200$, $a_{11} = 59.5$, $a_{22} = 60.5$, $a_{12} = 0$ (corresponding to a two-component ^{41}K condensate in a ring trap with $N = 4 \times 10^4$, trap circumference $L = 254 \mu\text{m}$, transverse frequency $\omega_{\perp} = 2\pi \times 1913 \text{ Hz}$, observation time $T = 24.9 \text{ ms}$, oscillator amplitude $\Omega = 2\pi \times 9.56 \text{ Hz}$, frequency $\omega = 2\pi \times 9.56 \text{ kHz}$ and modulation $\delta = 0.085$). **a**, Example of bubble formation: the spinor Bose field is initially in a false vacuum ($\tilde{t} = 0.9$, blue solid); quantum fluctuations cause the field to tunnel out ($\tilde{t} = 4.5$, green dash-dotted); three bubbles are formed in true vacua ($\tilde{t} = 9$, yellow dashed); they grow until one bubble meets another bubble in the second minimum, creating a domain wall ($\tilde{t} = 15$, pink dotted). **b**, Effective field potential (dotted lines mark the potential maxima). **c**, Time evolution of the relative number density difference p_z after a $\pi/2$ rotation, which converts the relative phase into a population difference.

space. The primary physical effect of the noise is to allow spontaneous tunneling and scattering processes that are disallowed in pure Gross-Pitaevskii theory. Quantum noise is added to the classical false vacuum state as $\psi(\mathbf{r}) = \psi_0(\mathbf{r}) + \sum_{j=1}^M \alpha_j \exp(i\mathbf{k}_j \mathbf{r}) / \sqrt{V}$. Here α_j are complex Gaussian variables with $\alpha_j^* \alpha_i = \delta_{ij}/2$, thus sampling fluctuations of the false vacuum. Quantum tunneling for a shallow potential well is equivalent to an activation process caused by the vacuum fluctuations of the quantum field, represented by the initial fluctuations of the Wigner phase-space representation.

The TWA is a truncation of the expansion in the powers of M/N up to and including the terms of order 1 [27]. Therefore, the number of modes M is chosen to represent the physical system, while being much smaller than the number of atoms N . For the 1D and the 3D simulations, $M/N \equiv N_{\text{grid}}^d / (\tilde{L}^d \tilde{\rho}_0) \approx 10^{-2}$, and for the 2D simulation $M/N \approx 3 \times 10^{-2}$, where N_{grid} is the number of grid points in one dimension.

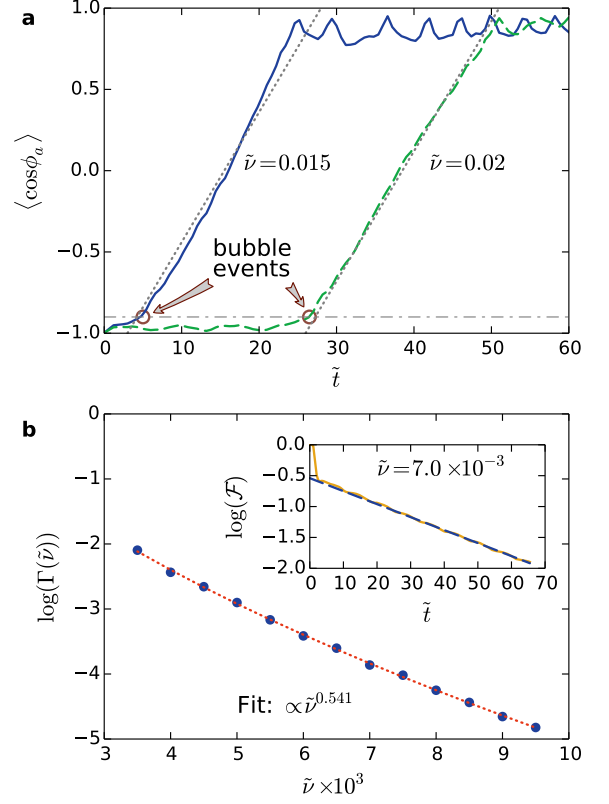


FIG. 2. Bubble nucleation probability. **a**, Single trajectories of the relative phase at different $\tilde{\nu}$, where $\langle \cos \phi_a \rangle = \frac{1}{L} \int_0^L dx \cos \phi_a(x)$. Initially the field is trapped in a false vacuum. A bubble appears via quantum tunneling at $t\omega_0 = 5$ (blue line), $t\omega_0 = 26$ (green line). The tunneling time is longer for larger couplings ν , consistent with dissipative slowing down of tunneling [28]. The bubble grows at the speed of sound (1 in our units). **b** (inset), Probability of bubble nucleation for $\tilde{\nu} = 8 \times 10^{-3}$ and its exponential fit. **b**, Dependence of the tunneling rate Γ on the coupling $\tilde{\nu}$ for $\lambda = 1.3$. This is extracted from the survival probability of the bubble nucleation, which behaves as $\mathcal{F} = \exp(-\Gamma t)$. For this relatively shallow effective potential, we extract that $\Delta B \propto \tilde{\nu}^{0.541}$.

We propagate this state in real time by solving the time-dependent coupled equations

$$i\hbar \partial_t \psi_j = \left[-\frac{\hbar^2}{2m} \nabla^2 - \mu + g \left(|\psi_j|^2 - \frac{M}{V} \right) \right] \psi_j - \nu_t \psi_{3-j}, \quad (6)$$

where index $j = 1, 2$, and with the coupling ν_t modulated in time.

The results of the simulations are shown in Fig. 1. The single trajectory dynamics shown here features the creation of three bubbles. Collisions of bubbles result either in the creation of localized long-lived oscillating structures known as oscillons [21, 31, 32], or domain walls if the colliding bubbles belong to topologically distinct vacua.

To quantify the tunneling process, we calculate the

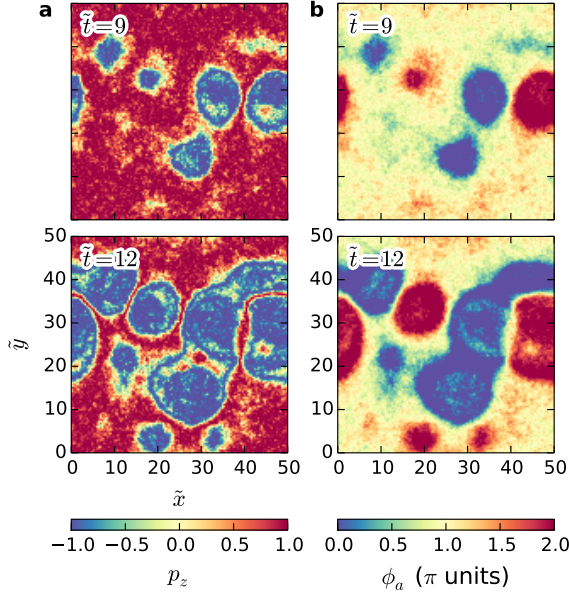


FIG. 3. Bubble formation in 2D. Snapshots representing the number density difference (a) and the phase difference (b) with $N_{\text{grid}} = 128$ and dimensionless parameters $\lambda = 1.1$, $\tilde{\omega} = 50$, $\tilde{\nu} = 0.005$, $\tilde{\rho}_0 = 200$, $a_{11} = 59.5$, $a_{22} = 60.5$, $a_{12} = 0$. Bubbles are seeded through quantum tunnelling and grow ($\tilde{t} = 9$) forming domain walls between bubbles with two distinct phases of 0 and 2π ($\tilde{t} = 12$).

probability that the system has not yet decayed at time t . At long time scales it should behave as $\mathcal{F} = \exp(-\Gamma t)$ [33], where Γ is the decay rate from the false vacuum. From the probability of bubble creation over time $\mathcal{P}(t)$, the “survival” probability can be calculated as $\mathcal{F} = 1 - \int_0^t \mathcal{P}(t') dt'$ and the decay rate can be extracted. In the weak tunneling limit it can be written in the form $\Gamma = A \exp(-B/\hbar)$ and the coefficients A and B were calculated in Refs. [1, 9] in limiting cases. Our numerical simulations are far from that to allow an experiment on reasonable time-scale. In the following we focus on the damping term, the third term in Eq. (5), which suppresses tunneling leading to a correction to B , $\Delta B \propto \tilde{\nu}^{3/2}$ [28] in the weak tunneling regime.

Our TWA approach is expected to yield accurate predictions for the relatively shallow effective potentials necessary for tunneling over laboratory time-scales [10]. In Fig. 2 we present the scaling of the tunneling rate of bubbles for $\lambda = 1.3$. We find that the corresponding $\Delta B \propto \tilde{\nu}^{0.5}$. We checked numerically that by increasing λ the exponent also increases. However, our approach is not valid for larger values of λ , and this is also less accessible experimentally. The observed behavior provides strong evidence of a quantum tunneling process.

For implementing an analog quantum simulation of the false vacuum we propose to use a spinor condensate with a suppressed inter-state scattering length a_{12} . As

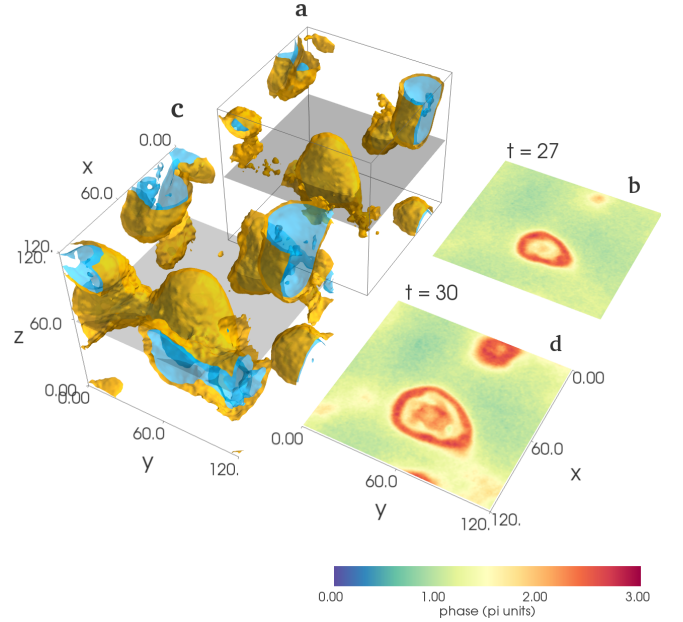


FIG. 4. Bubble formation in 3D. Simulation of a single three-dimensional random trajectory with $N_{\text{grid}} = 128$ and dimensionless parameters $\lambda = 1.01$, $\tilde{\omega} = 50$, $\tilde{\nu} = 0.05$, $\tilde{\rho}_0 = 100$, $a_{11} = 59.5$, $a_{22} = 60.5$, $a_{12} = 0$. This corresponds to a two-component ^{41}K condensate with realistic parameters in a uniform box trap [34] with $N = 1.72 \times 10^8$, trap size $L = 95.8 \mu\text{m}$, observation time $T = 7.4 \text{ ms}$, oscillator amplitude $\Omega = 2\pi \times 96.9 \text{ Hz}$, frequency $\omega = 2\pi \times 43.3 \text{ kHz}$ and modulation $\delta = 0.16$. a, c, The 3D outline of relative phases of 0.2 (yellow) and 0.8 (blue) converted into a relative number density difference p_z . b, d, The phase difference in a 2D slice to show the internal phase variation near the bubble walls (location is marked with gray on the corresponding 3D plots). Dimensionless time is $\tilde{t} = 27$ for a, b, and $\tilde{t} = 30$ for c, d.

an example of this, a two-component condensate [12] of ^{41}K atoms prepared in two Zeeman states $|1\rangle = |F = 1, m_F = 1\rangle$ and $|2\rangle = |F = 1, m_F = 0\rangle$ is predicted to have an inter-state Feshbach resonance with $a_{12} \approx 0$ at 675.3 G [35]. States $|1\rangle$ and $|2\rangle$ are separated by 61.93 MHz at this magnetic field and are coupled via a magnetic dipole transition. The two intra-state scattering lengths are $a_{11} = 59.5a_0$ and $a_{22} = 60.5a_0$, where a_0 is the Bohr radius. The stretched state $|1\rangle$ will be condensed in the optical dipole trap. The s-wave scattering length a_{11} has a favourable value for the fast thermalization process and is not too large to introduce inelastic losses. A pulsed radiofrequency field of 51.63 MHz will generate the 50:50 superposition of two states $|1\rangle$ and $|2\rangle$.

A toroidal or flat linear atom trap with tight transverse confinement ($\sim 10 \text{ kHz}$) will provide the 1D system with the desired initial uniform distribution of the atom density along the axial coordinate. Simulation data shown in Fig. 1 and Fig. 2 corresponds to a 1D toroidal trap. A similar experiment is also feasible in 2D or 3D, using

experimentally realized 2D [36] and uniform 3D [34] trapping potentials. The results of the corresponding simulations with realistic experimental parameters are shown in Figs. 3 and 4. The 2D simulations of Fig. 3 show the nucleation of near spherical bubbles and demonstrate both the formation of domain walls and 2D oscillons, i.e. long-lived localized non-topological structures. The 3D simulations reveal even more complex dynamics with multiple nested bubbles having novel topological structure seen in Fig. 4a. More detailed simulations and results from prospective experiments may further elucidate the nature of complex bubble structures and questions like the prevalence of asymmetry in bubble creation.

Demonstrating the false vacuum decay by quantum tunneling will pave the way to analog quantum simulations of a cosmological process that is currently not accessible to exact computer simulation. Combined with accurate observational data of the correlations in the cosmic microwave background, this may eventually help us to refine cosmological models and answer the question “Where do we come from?”

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